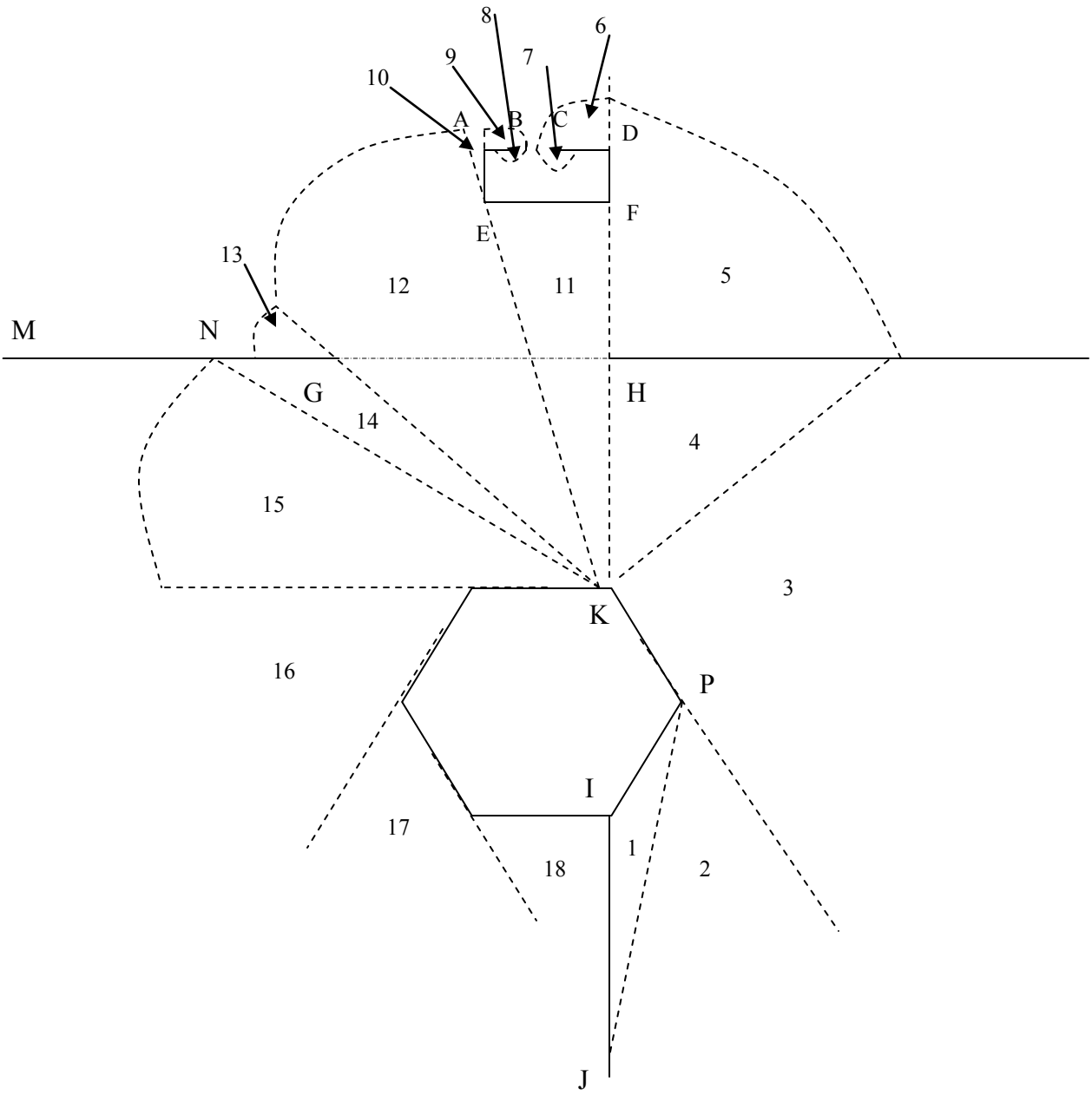
The Great Hungry Goat Problem - 2011 *St. Louis Confluence*

Solution

There are 18 regions to consider. The tether from K is 20. KH=GH=10. HF=EF=5. AE=DF=2.5. AB=1. CD=2. D, F, H, K, I, and J are collinear and perpendicular to the fence MNH. The fence MNH is also parallel to the side of the hexagonal barn. Use 3.14159 for π. Estimate areas to three decimal places.



Region 1: Area of Triangle
With an exterior auxiliary 30-60 rt triangle using IP as the hypotenuse, height =2.5. By Pythagorean Thm. Large base of 14.79 - smaller base of exterior auxiliary triangle = base of 10.46.
Area $= 1/2\ (2.5)(10.46) \approx 13.075$ sq units

Region 2: Area of Sector
Radius = 15.
By Law of Sines, angle P of triangle PIJ in Region 1 equals 20.41°. Thus the sector angle = 60-20.41° = 39.59°.
Area = $39.59/360\ (\pi)(15^2) \approx 77.734$ sq units.

Region 3: Area of Sector
Radius = 20.
Sector angle = 90°.
Area = $1/4\ (\pi)(20^2) \approx 314.159$ sq units.

Region 4: Area of Triangle 30-60 rt triangle with sides 10, $10\sqrt{3}$, and 20.
Area = $1/2\ (10)(10\sqrt{3}) \approx 86.603$ sq units.

Region 5: Area of Sector
Radius = 10.
Area = $1/4(\pi)(10^2) \approx 78.540$ sq units.

Region 6: Area of Sector
Radius = 2.5.
Area = $1/4\ (\pi)(2.5^2) \approx 4.909$ sq units.

Region 7: Area of Semicircle
Radius = 0.5.
Area = $1/2\ (\pi)(0.5^2) \approx 0.393$ sq units.

Region 16: Area of a Sector
Radius = 15.
Sector angle = 60°.
Area = $1/6\ (\pi)(15^2) \approx 117.810$ sq units.

Region 17: Area of Sector
Radius = 10.
Sector angle = 60°.
Area = $1/6\ (\pi)(10^2) \approx 52.360$.

Region 18: Area of Sector
Radius 5.
Sector angle = 60°.
Area = $1/6(\pi)(5^2) \approx 13.090$ sq units.

Region 15: Area of Sector - Clockwise from K
Radius = 20.
Triangle KHN is a 30-60 rt triangle creating congruent alternate interior angles of 30°.
Sector angle = 30°.
Area = $30/360(\pi)(20^2) \approx 104.720$ sq units.

Region 14: Area of a Triangle
Area (KNG) = area(KNH) - area(KGH).
Triangle KGH is a 45-45 rt triangle.
Area = $1/2(10)(10\sqrt{3}) - 1/2(10)(10)$
Area ≈ 36.603 sq units.

Region 13: Area of Sector
Radius = $20—10\sqrt{2} \approx 34.340$ units.
Sector angle = 45°.
Area = $45/360(\pi)(34.340) \approx 13.485$ sq units.

Region 11: Area of Triangle
Area = $1/2(5)(5) = 37.500$ sq units.
We will need the angle at K for Region 12.
Angle K = $\tan^{-1}(5/15) \approx 18.43°$.
Hypotenuse = 15.81 by Pythagorean Thm.

Region 12: Area of Sector
Radius 20.
Sector angle $\approx 45° - 18.43° \approx 26.57°$
Area = $26.57/360\ (\pi)(20^2) \approx 92.747$ sq units.

Region 10: Area of Sector
Radius $\approx 20 - 15.81 \approx 4.19$.
Sector angle $\approx 18.43°$ corresponding angles
Area = $18.43/360\ (\pi)(4.19^2) \approx 2.824$ sq units.

Region 9: Area of Quarter Circle
Radius $\approx 4.19 - 2.5 \approx 1.69$
Area = $1/4\ (\pi)(1.69^2) \approx 2.243$ sq units.

Region 8: Area of a Semi-Circle
Radius $\approx 1.69 - 1.0 \approx 0.69$
Area $= 1/2\ (\pi)(0.69^2) \approx 0.748$ sq units.

TOTAL of all 18 regions

1,049.543 sq units